# CHANGE DETECTION IN FULL AND DUAL POLARIZATION SAR DATA AND THE COMPLEX WISHART DISTRIBUTION

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## ABSTRACT

A test statistic for equality of two complex variance-covariance matrices following the complex Wishart distribution with an associated probability of observing a smaller value of the test statistic is sketched. We demonstrate the use of the test statistic and the associated probability measure for change detection in both full and dual polarimetry synthetic aperture radar (SAR) data collected by the Danish EMISAR system.

## INTRODUCTION

In (1) we introduced a new test statistic for equality of two variance-covariance matrices following the complex Wishart distribution with an associated probability of observing a smaller value of the test statistic. We also demonstrated its use for change detection in both fully polarimetric and azimuthal symmetric synthetic aperture radar (SAR) data (2). An appendix in (1) gave details and mentioned the possibility of application of the test statistic and the associated probability to block-

*Figure 1: L-band RGB image (R is HV, G is HH and B is VV), 17 April 1998; 5,120 m by 5,120 m.*

structured covariance SAR data. This feature was further described in (3), which also showed the application of the test statistic and the associated probability to edge detection in polarimetric SAR data. At that time, i.e., in the early 2000s, not many workers had access to polarimetric SAR data. With the advent of several spaceborne polarimetric SAR instruments

- the Japanese ALOS (a.k.a. DAICHI), L-band with single, dual and full polarization, (ALOS completed its operations in May 2011 and will be followed by ALOS-2),
- the Canadian Radarsat-2, C-band with single, dual and full polarization,
- the German TerraSAR-X, X-band with single, dual and full polarization,
- the Italian COSMO-SkyMed, X-band with single and dual polarization,
- the European (ESA) Sentinel-1, C-band with single and dual polarization (to be launched in 2013),

this situation has changed. For power supply and coverage reasons the instruments among the above which have full polarization capability are mostly operated in (either single or) dual polarization modes, i.e., they transmit one polarization only and receive (either one or) both polarizations. We therefore think that a revisit to the methods described and sketched in (1, 3), with emphasis on dual polarization SAR data, is timely and this contribution sketches the theoretical results and gives case studies for both full and dual polarimetry data.

## METHODS

In the covariance representation of multi-look polarimetric SAR data, a test statistic for equality of two matrices $X$ and $Y$ following the complex Wishart distribution is

*Figure 2: L-band RGB image (R is HV, G is HH and B is VV), 16 August 1998; 5,120 m by 5,120 m.*

$$Q = \frac{(N+M)^{p(N+M)}}{N^{pN} M^{pM}} \frac{|X|^N |Y|^M}{|X+Y|^{N+M}}$$

where *N* is the number of looks for *X*, *M* is the number of looks for *Y*, *p* is the size of the matrices (2 for dual and 3 for full polarization data), and |…| denotes the determinant of the matrix. The test statistic *Q* lies in the interval [0,1] and is equal to 1 for equality.

The probability of obtaining a smaller value of $-2\rho \ln Q$ is

$$P\{-2\rho \ln Q \le z\} \simeq P\{\chi^2(p^2) \le z\} + \omega_2[P\{\chi^2(p^2+4) \le z\} - P\{\chi^2(p^2) \le z\}]$$

where

$$\rho = 1 - \frac{2p^2-1}{6p}\left(\frac{1}{N} + \frac{1}{M} - \frac{1}{N+M}\right)$$

and

$$\omega_2 = -\frac{p^2}{4}\left(1 - \frac{1}{\rho}\right)^2 + \frac{p^2(p^2-1)}{24\rho^2}\left(\frac{1}{N^2} + \frac{1}{M^2} - \frac{1}{(N+M)^2}\right).$$

For details on for example the azimuthal symmetry and the diagonal cases, see (1).

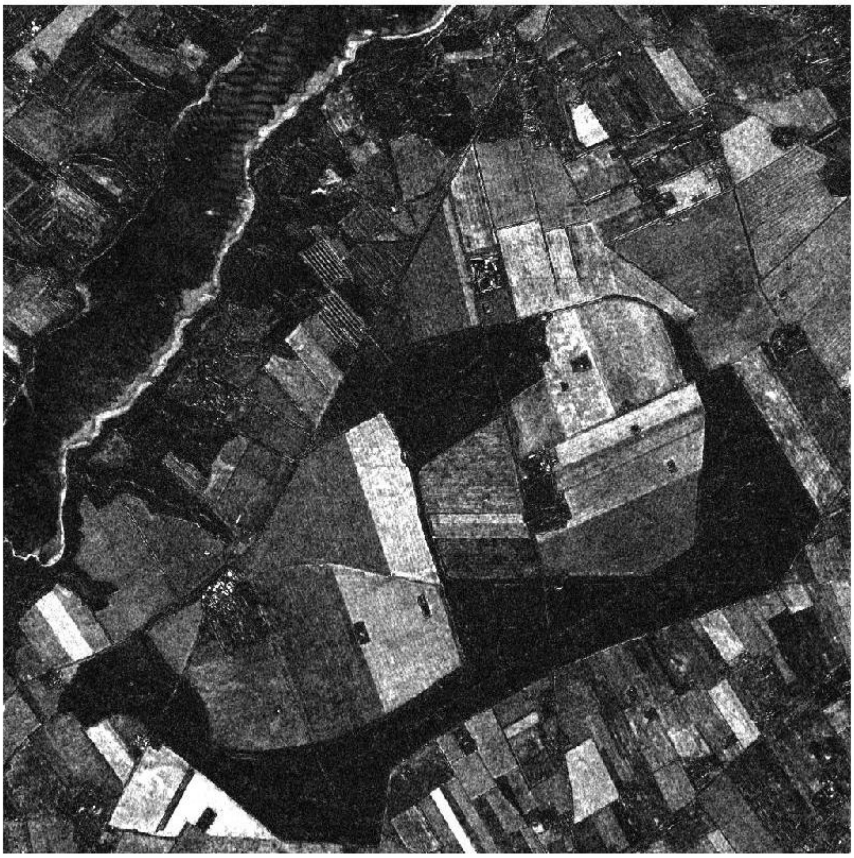

*Figure 3: $-2\rho \ln Q$, full polarization.*

## RESULTS

We apply the above test statistic and the associated probability measure to full and dual polarization data from the Danish EMISAR system (4, 5). Figure 1 shows a full polarimetry, multi-look ($N$=13) L-band image from an agricultural site near Foulum, Denmark acquired on 17 April 1998. The image consists of 1,024 by 1,024 5 meter pixels. Figure 2 shows a full polarimetry, multi-look ($M$=13) L-band image from the same area acquired on 16 August May 1998. Figure 3 shows $-2\rho \ln Q$ and Figure 4 shows the probability of obtaining a smaller value of $-2\rho \ln Q$. This is the probability of change. Figures 5 and 6 show the same quantities for a dual polarization case (HH and HV).

We clearly see that virtually the same changes are found in the two cases, yet the strength of the detection seems slightly higher for the fully polarimetric case compared to the dual-polarisation case. The conclusion may, of course, be different for other targets and changes.

## SOFTWARE

Three Matlab programs will be made available: wishart_det, wishart_change and freadenvisar.

wishart_det calculates determinants of complex variance-covariance matrices and flags singularity in the following cases

- full polarimetry (including azimuthal symmetry and diagonal only),
- dual polarimetry (including diagonal only).

*Figure 4: Probability of obtaining smaller value of $-2\rho \ln Q$, i.e., change probability, full polarization.*

*Figure 5:* $-2\rho \ln Q$*, dual polarization, HH and HV.*

wishart_change tests for equality of two complex variance-covariance matrices in the following cases

- full polarimetry (including azimuthal symmetry and diagonal only),
- dual polarimetry (including diagonal only), and
- single channel only.

freadenvisar reads covariance matrix polarimetric SAR data (with an ENVI-like header file) from disk and stores them in the order needed for the change detection programs.

Software in ENVI/IDL and Python will be made available also.

## CONCLUSIONS

Unlike most other change detection methods for polarimetric SAR data, the proposed method uses the polarimetric information in a unified way. Future work will include 1) complex Wishart distribution based segmentation of the images followed by change detection in the segments and, 2) the extension of the test statistic to comparison of several (more than two) matrices which can be applied for change detection in truly multi-temporal polarimetric SAR data.

*Figure 6: Probability of obtaining smaller value of* $-2\rho \ln Q$*, i.e., change probability, dual polarization, HH and HV.*

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